

TESTING VOLATILITY PERSISTENCE WITH FRACTIONAL INTEGRATION AND COINTEGRATION IN WORLDWIDE COMMODITY MARKETS

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Abstract

In this paper, we examine the volatility of commodity prices around the world using monthly data for the time period 1960–2022. During this period, there were significant economic crises that increased the prices of both energy and non-energy commodities in the short term. These past crises may provide insights into the behavior of current crises, as they generally exhibit similar patterns. We use fractional integration methods and find that the volatility for each variable exhibits mean reversion, with the effect of the shocks disappearing in the long run. While this may seem obvious, our paper is the first to empirically demonstrate this significant process of convergence using a flexible time series model. As an additional contribution, we complement our analysis by incorporating a fractional cointegration test to examine potential relationships among the commodity prices. Our findings reveal the existence of four distinct cointegrating relationships within the set of ten commodity prices. This implies that these commodities are not entirely independent and may share underlying connections that contribute to their price movements over time. This valuable insight further enriches our understanding of the intricate interactions within the commodity market. We conclude that although countries do not have effective policies for mitigating volatility, it is only a short-term phenomenon that will disappear in the long term.

Keywords: volatility; commodity crisis; commodity prices; persistence; fractional integration; fractional cointegration.

JEL Classification: C22; C12; Q02; C32.

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1. INTRODUCTION

Global macroeconomic shocks have been a significant source of commodity price volatility since the stagflation of the 1970s. Commodity prices refer to the market prices of raw materials or primary agricultural products such as oil, gold, copper, wheat, etc. These prices are determined by supply and demand factors, as well as by geopolitical events, weather conditions, and other factors. Commodity prices can have a significant impact on the global economy, as they can affect the cost of production for businesses and the prices consumers pay for goods and services. In addition, when commodity prices rise, countries that are net exporters of these commodities can experience economic growth. Overall, commodity prices can be an important indicator of the health of the global economy, and fluctuations in these prices can have significant impacts on businesses, consumers, and entire countries.

Decades ago, several economic crises were triggered by spikes in commodity prices, and the current economic scenario exhibits some similar patterns of behavior. One primary cause of variability in global commodity prices was the oil price shocks of the 1970s and 1980s, which led to significant economic dislocations and geopolitical instability. Today, in 2023, we are witnessing a similar trend, with the prices of many commodities, including oil and gas, among others, experiencing significant increases. As a result, there is significant inflation in all products around the world. The increase in prices cannot be attributed to a single factor, but rather is the result of a complex interplay of multiple reasons.

There are several significant reasons behind the increase in commodity prices. First, the COVID-19 pandemic has disrupted global supply chains, leading to shortages and surpluses in certain commodities. Second, geopolitical tensions between countries or regions can impact the supply of commodities. Third, increasing demand from emerging markets such as China and India have led to price increases for commodities due to competition for limited supplies. Finally, the ongoing war between Russia and Ukraine has caused disruptions to the supply of natural gas and electricity, resulting in sharp price increases in the global energy markets. The complexity of these factors highlights the critical concern of commodity price volatility for economists. So far, there has been limited research on the topic of commodity price volatility, with only a few papers exploring this issue. However, this is a topic that deserves further investigation, given the significant impact that volatility in commodity prices have on economies around the world.

The objective of this study is to analyze volatility in global commodity markets. The commodity prices we consider are energy and non-energy commodities (precious metals, metals, and minerals). Specifically, we aim to examine the flow of volatility

over the time period of our sample (1960-2022). Another objective, which further complements the volatility analysis, is to examine the potential cointegration relationships that may be present between commodities. To ensure that our study is innovative and has not been explored before, we conducted a review of the literature. We find that this idea has not been explored using an updated sample, as we have done, nor has it been examined using flexible methods like fractional integration and cointegration.

Following, among others, Granger and Ding (1995), we investigate volatility using proxies such as absolute returns and squared returns. They suggest these two measures might present some degree of long memory behavior. Therefore, we estimate the order of integration of the series to test if long memory holds and if mean reversion takes place in the data.

This paper contributes to the existing limited of literature on commodity market volatility by presenting several novel insights and offering evidence of fractional cointegrating relationships among these commodity prices. This evidence strongly suggests a connection among various worldwide commodities. To the best of our knowledge, this study is also the first to comprehensively investigate volatility persistence in worldwide commodity markets using fractional integration, while simultaneously examining potential relationships among these commodity prices through fractional cointegration within sample 1960-2022. This dual approach enhances the understanding of both the volatility of the worldwide commodity market and the interconnectedness of these commodities.

We use a simple version of a testing procedure of Robinson (1994), which permits us to test unit roots in potentially fractional contexts with a standard null limit distribution, which is unaffected by the inclusion or not of deterministic trends. To complement the aforementioned approach, we employ the fractional cointegrated vector autoregressive (FCVAR) methodology developed by Johansen (2008) and subsequently refined by Johansen and Nielsen (2010, 2012, 2014) to address the second aspect related to the cointegration relationships. Specifically, this study makes a fourfold contribution.

(i) First, it applies fractional integration techniques to provide evidence of the stochastic properties of prices and volatility proxies. This approach is more general than the standard $I(0)/I(1)$ specifications, as it allows for fractional values of the integration parameter, avoiding restrictive assumptions on the dynamic behavior of individual series.

(ii) Second, regarding the findings in the context of commodity prices, our analysis indicates that only the crude oil of Dubai exhibits mean reversion. This implies that, unlike other commodity markets, the price of oil in Dubai tends to return to its

long-term mean after experiencing short-term fluctuations. The case of Dubai is exceptional, as its economy is stable and it has a large reserve of oil, which makes its production less dependent on fluctuations in the global market and less sensitive to economic crises and price fluctuations. These factors have collectively contributed to Dubai's resilience and prosperity in the face of global economic uncertainties.

(iii) Third, our analysis shows that all the volatility proxies in commodity markets, such as absolute and squared returns, exhibit mean reversion. This finding expands upon the existing literature on volatility persistence in commodity markets, as it suggests that even highly volatile markets tend to revert to their mean values over time. This indicates that they tend to return to their long-term average after experiencing short-term fluctuations, which is a normal phenomenon in the markets as volatility cannot persist indefinitely. This suggests that any periods of high volatility caused by crises or global catastrophes are temporary and will eventually disappear.

(iv) Our findings reveal the presence of four distinct cointegrating relationships among the ten commodity prices. This suggests that these commodities, despite being situated in different parts of the world, are not entirely isolated entities; instead, they appear to share underlying connections that contribute to their price fluctuations over time on a global scale. This significant revelation deepens our understanding of the intricate interplays within the domain of commodity trading.

The rest of the article is structured as follows: *Section 2* provides a brief literature review of papers related to commodity market volatility. *Section 3* describes the data used in our analysis. In *Section 4*, we outline the methodology employed. *Section 5* presents the results and discussion. Finally, *Section 6* concludes the research.

2. BRIEF LITERATURE REVIEW

This first part of the review focuses on commodity price volatility. We have selected prominent investigations, starting with the seminal work of Samuelson (1965), who was the first to find that commodity prices fluctuate randomly. Over time, other studies pointed in a similar direction. Understanding the nature of this stochastic behavior is crucial for commodity markets. Pindyck (2001) used the unit root test to assess crude oil, gas, and coal data for the period 1870-1996 (in yearly data) for the U.S. dataset. He shows that the price mean reverts to stochastically varying trend lines. Beck (2001) investigated volatility clustering in commodity price distributions. The data used included 20 commodity prices from 1840 to 1996. The kind of method used to analyze this issue was GARCH-type models and it was found that changes in the expected price variance do not significantly affect pricing.

Geman (2007) investigated energy commodity prices of oil and gas. The sample spanned 13 years, from 1994-2007. The Dickey and Fuller (ADF, 1979) and Phillips-Perron (PP, 1988) tests were utilized. The study found evidence of mean reversion in the sample up to the year 1999 for the two variables. However, starting from 2000, there has been volatility and a transition into a random walk.

Jacks *et al.* (2011) analyzed commodity prices from around the world, in categories of poor and rich countries. The timeline of the data analyzed is from 1700 to 2000 and employed a GARCH model. They discover a slight trend since 1700, also finding that the commodity prices are more volatile than the manufacturer's ones. Brooks and Prokopczuk (2013) studied the stochastic behavior of the prices and volatilities of a sample of the six most important commodity markets. The data they used was from the U.S. The commodities include crude oil, gasoline, gold, silver, soybeans, and wheat (these last two are from agricultural commodities markets). They used a Markov chain Monte Carlo estimation approach based on Bayesian methods, finding that the commodities can be a useful diversifier of equity volatility as well as equity returns. Cavalcanti *et al.* (2015) investigated the impacts of the growth and volatility of commodity prices. Their variables were GDP, determinants of growth and the Commodity Terms of Trade (also known as CToT Index) based on 32 primary commodities. The period spanned from 1970 to 2007 for a list of 118 countries. The methodology used was the GMM approach and the dynamic common correlated effects pooled mean group. Their findings showed that commodity price volatility has a negative impact on economic growth, but it has no impact on productivity. Joets *et al.* (2017) investigated the impact of macroeconomic uncertainty on commodity price volatility. The data they considered were the 19 principal commodity markets: energy, precious metals, agriculture, and industry. The time period started in 1976 and ended in April 2015 with monthly data. The structural Threshold Vector Autoregressive (TVAR) model was employed, and evidence was found to suggest that uncertainty episodes are not necessarily accompanied by high volatility in commodity prices.

Bakas and Triantafyllou (2020) demonstrated for the first time that pandemic uncertainty shocks have a notable and adverse impact on commodity volatility. Employing VAR models, the study analyzed excess returns of S&P GSCI broad and sub-indexes of gold and crude oil, covering the time frame between January 1996 and March 2020. The study found that higher uncertainty about a pandemic is transferred to economic agents rather than to uncertainty about aggregate demand and supply conditions. In other words, the pandemic uncertainty shocks reduce commodity price volatility via the disruption in global demand during pandemic times.

On the other hand, other authors have investigated commodity prices but focusing on price jumps, and asymmetry, and obtaining evidence that is unrelated to volatility. Thus, for example, Chevallier and Ielpo (2014) discovered that commodity

market jumps are more common than in other market classes, while there is a high discrepancy within the commodity markets concerning the number, size and sign of the jumps. Prokopczuk *et al.* (2016) present evidence of jumps in four energy markets and show that modeling jumps do not provide any significant improvement for volatility forecasting. Other authors looking at similar issues include Diewald *et al.* (2015), Lombardi and Ravazzolo (2016), Ohashi and Okimoto (2016), and Nguyen and Prokopczuk (2019).

Other authors have focused on commodity prices exhibiting asymmetric adjustments, such as Cashin *et al.* (2002), who were the first that analyzed these asymmetry adjustments. Ghoshray (2019) found that commodity prices are stationary, also obtaining evidence of asymmetry in the behavior of commodity prices. In addition, this author suggests that in future studies, commodity prices should be modeled on an individual basis rather than an aggregate level.

Commodity prices are strongly heteroskedastic (see, e.g., Duffie *et al.* 1999) and are extremely volatile. However, there has not been much research done on commodity price volatility. Studying commodity prices without considering volatility is a limitation that we have endeavored to resolve in this work. For this purpose, we propose a flexible time series approach that allows for fractional degrees of differentiation in the commodity absolute and squared price returns.

This second part of the review focuses on cointegration relationships among commodity prices. After delving into the literature, we have not come across studies that address fractional cointegration among these commodity markets for this specific and novel sample. The only studies we have been able to find, among others, are those that solely analyze cointegration without delving into the flexibility offered by fractional cointegration.

Hagen (1989) employed cointegration to analyze primary commodities through their relative prices in a time series dataset spanning from 1900 to 1986 and discovered cointegration among them. Karbuz and Jumah (1995) examined long-run cointegration relationships among different commodity exchanges (cocoa and coffee prices). They generally found that these commodities tend to move in tandem over the long term and are efficiently arbitrated in the international market. Ciaian (2011) focused solely on commodity prices related to energy, bioenergy, and food prices. They conducted a time series analysis for a sample period from 1994 to 2008 and concluded that crude oil and agricultural commodity prices are interdependent. Nazlioglu and Soytaş (2012) explored the relationships between world oil prices and other agricultural commodity prices using panel cointegration and Granger causality techniques on a monthly dataset spanning from 1980 to 2020. They uncovered compelling evidence of the impact of world oil prices.

Casoli and Lucchetti (2022) introduced a cointegration approach based on permanent and transitory decomposition. As an application of this technique, they analyzed a set of commodity prices to assess the co-movement among various markets. Their findings suggested that commodity prices indeed move together.

3. DATA

The data were obtained from the *World Bank Commodity Price Data* for ten variables, composed of both energy and non-energy commodities. The variables are prices of the different commodity markets. The variables and their unit of measure in brackets () are as follows: The crude oil on average (\$/bbl), crude oil of Brent (\$/bbl), crude oil of Dubai (\$/bbl), natural gas of US (\$/mmbtu), natural gas of EU (\$/mmbtu), aluminium (\$/mt), Iron ore cfr spot (\$/dmtu), copper (\$/mt), gold (\$/troy oz) and silver (\$/troy oz).

The timeline of the data starts in January 1960 and ends in September 2022 for all the variables, and the frequency of the data is monthly. Therefore, we have a total of 753 observations. In *Table 1*, we describe in detail our data composition.

Table 1

Data composition

Energy	Non-Energy commodities	
	Precious metals	Metals and minerals
Natural gas EU	Gold	Aluminium
Natural gas US	Silver	Copper
Oil AV		Iron
Oil Brent		
Oil Dubai		

The commodity price series are displayed in *Figure 1*. It can be observed that the series exhibit some deviations and are clearly nonstationary. To conduct the integration analysis, we used the logarithm to mathematically comprise the series. *Figure 2* shows in a single graph, the returns of the price series, which is the growth rate of the commodity prices. As can be seen in the graph, they are all apparently stationary. Note that for the proxies of volatility, we use the absolute value and squared values of the growth rate (for more detail about their creation, see the Methodology section).

Figure 1. Commodity prices graph (raw series)

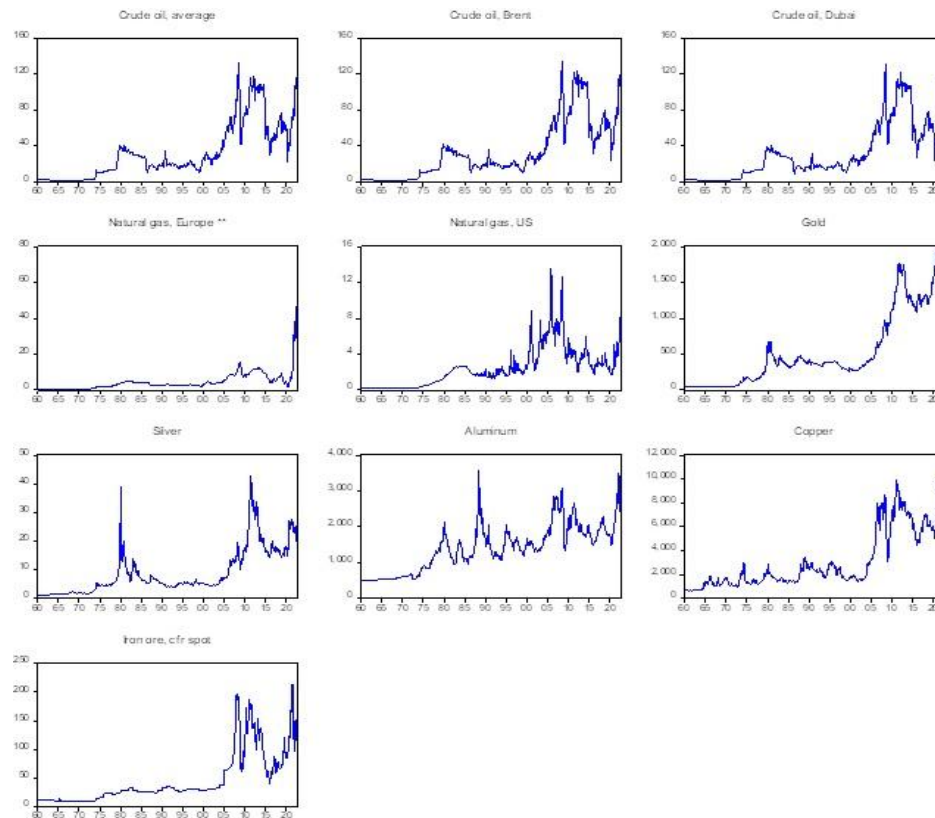
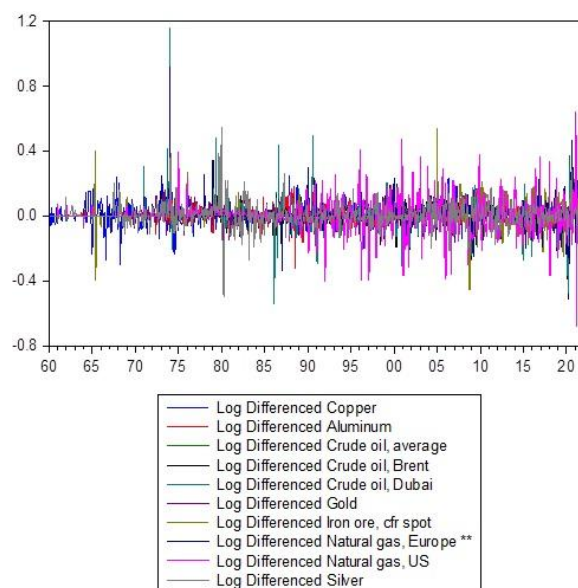


Figure 2. Time series plot of returns*



4. METHODOLOGY

Fractional Integration

Fractional integration is the (time series) technique employed in this paper. It means that fractional differentiation might be required in a series to render it stationary $I(0)$. A process is said to be integrated of order 0 or $I(0)$ if it is covariance stationary and the infinite sum of its autocovariances is finite. Within this category we include the stationary and invertible AutoRegressive Moving Average class of models. If d differences are required, the series is then said to be integrated of order d , or $I(d)$, expressed in mathematical notation as

$$(1 - B)^d x_t = u_t, \quad t = 0, \pm 1, \dots, \quad (1)$$

where B refers to the backshift (also known as the lag, L) operator ($B^n x_t = L^n x_t = x_{t-n}$) and u_t is $I(0)$. If $d > 0$, x_t displays the property of long memory since the infinite sum of the autocovariances is infinite, and the spectral density function (i.e., the Fourier transform of the autocovariances) is unbounded at the zero frequency. Using a binomial expansion, the polynomial in B in (1) can be expressed as

$$(1 - B)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} B^j,$$

where $\Gamma(x)$ is the Gamma function, or alternatively as

$$(1 - B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} B^2 - \dots$$

and thus, x_t can be expressed in terms of all its history.

In the empirical application carried out in the following section, we estimate the differencing parameter by using the Whittle function expressed in the frequency domain. For this purpose, we use a simple version of a testing procedure developed in Robinson (1994) and widely used in empirical applications in many different fields.²

² See Gil-Alana and Robinson (1997) (economics); Abbritti *et al.* (2006, 2013) (finance); Perez de Gracia *et al.* (2014) (tourism), Barani *et al.* (2021) (syismography), Li *et al.* (2021) (hydrology), etc.

Fractional Cointegration

Cointegration methods have been popular tools in time series work since their introduction around 35 years ago. Engle and Granger (1987) were the first to formalize the idea of integrated variables sharing an equilibrium relationship, which turned out to be either stationary or have a lower degree of integration than the original series in the single-equation model. They termed this relationship property cointegration, signifying co-movements among trending variables that could be exploited to test for the existence of equilibrium relationships within a fully dynamic specification framework.

The multivariate generalization of Engle and Granger (1987) is a procedure developed by Johansen (1988, 1991, 1995) and Johansen and Juselius (1990) within the framework of the vector autoregressive model (VAR). However, the VAR has less flexibility than the FCVAR because the memory parameter, d , in the fractional cases could be a non-integer, which helps explore the possible relationships between the variables.

The fractionally cointegrated vector autoregressive (FCVAR), by Johansen (2008) and Johansen and Nielsen (2010, 2012, 2014) is given in error correction form as follows:

$$\Delta^d X_t = \alpha \beta' \Delta^{d-b} L_b X_t + \sum_{i=1}^k \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t, \quad (2)$$

where

- $\varepsilon_t \sim i.i.d(0, \Omega)$, and Ω is the covariance matrix,
- d is the memory parameter,
- b is the degree of cointegration,
- Δ^d is the fractional difference operator,
- $L_b = 1 - \Delta^b$ is the fractional lag operator,
- α and β are the long-run parameters, as in Johansen (1995), $p \times r$ matrices with $0 \leq r \leq p$, where r is the cointegration rank or co-fractional rank, and p is the number of variables (dimension). Columns in β constitute the r cointegration vectors such that $\beta' X_t$ of order $d - b$ are the long-run equilibrium relations. The parameters in α are the adjustment coefficients, which represent the speed of adjustment towards equilibrium.
- Γ_i : short-run dynamics of the variables.

Two restrictions are used in their asymptotic analysis. Johansen and Nielsen (2010, 2012, 2014) employ two restrictions, namely $d \geq b$ and $d - b < \frac{1}{2}$. The inclusion of deterministic terms in the above FCVAR model (2) by Johansen and Nielsen (2012,

2014) is to implement the level parameter (denoted by the authors as μ) in the model. It is defined that $\rho' = \beta'\mu$, however, there is no unrestricted constant. This new model is as follows:

$$\Delta^d(X_t - \mu) = \alpha\beta'\Delta^{d-b}L_b(X_t - \mu) + \sum_{i=1}^k \Gamma_i \Delta^d L_b^i(X_t - \mu) + \varepsilon_t, \quad (3)$$

The model given in (3) is the most used in empirical analysis, with a level parameter included that impose $d = b$). Once the FCVAR is estimated, the cointegration test is conducted, which follows the same sequence as the VAR.

5. RESULTS AND DISCUSSION

We estimate the following regression model:

$$y_t = \alpha + \beta t + x_t, \quad (1 - B)^d x_t = u_t, \quad t = 1, 2, \dots \quad (4)$$

where u_t is $I(0)$ or a short-memory process.

Table 2 displays the estimates of the differencing parameter d along with the 95% confidence bands for the differencing parameter for three different specifications, namely i) with no deterministic terms, i.e., imposing $\alpha = \beta = 0$ in (4); ii) with a constant, i.e., $\beta = 0$ in (4); and iii) with a constant and a linear time trend. The coefficients marked in bold are those from the model selected in each case based on the statistical significance of the regressors. It is assumed that the error term u_t in (4) is autocorrelated. However, instead of imposing a standard ARMA model specification we follow the exponential spectral approach of Bloomfield (1973), which is very suitable in the context of long memory models as is the one used in this paper. These processes were first suggested by Granger (1980, 1981), Granger and Joyeux (1980), and Hosking (1981), and were justified by using aggregation of heterogeneous processes in Robinson (1978) and Granger (1980). However, the suspicions around long memory started many years ago with a pattern in the peaks of the spectrum, as identified in Granger (1966).

Table 2

Estimates of d for the logged price series

Series	No terms	An intercept	An intercept and a linear time trend
Aluminium	0.99 (0.94, 1.05)	1.17 (1.10, 1.23)	1.17 (1.10, 1.23)
Copper	1.00 (0.95, 1.06)	1.24 (1.16, 1.33)	1.24 (1.16, 1.33)
Gold	1.00 (0.95, 1.06)	1.17 (1.11, 1.24)	1.17 (1.11, 1.24)
Iron	1.00 (0.94, 1.06)	1.10 (1.04, 1.18)	1.10 (1.04, 1.18)
Natural gas EU	1.12 (1.06, 1.19)	1.14 (1.08, 1.20)	1.14 (1.08, 1.20)
Natural gas US	0.97 (0.91, 1.04)	0.97 (0.91, 1.03)	0.97 (0.91, 1.03)
Oil AV	1.12 (1.06, 1.18)	1.14 (1.07, 1.21)	1.14 (1.07, 1.21)
Oil Brent	1.11 (1.05, 0.99)	1.12 (1.06, 1.20)	1.12 (1.06, 1.20)
OilDubai	0.91 (0.85, 1.23)	0.91 (0.85, 0.99)*	0.91 (0.85, 0.99)
Silver	1.05 (1.08, 1.04)	1.15 (1.08, 1.23)	1.15 (1.08, 1.23)

In red and with an asterisk, evidence of mean reversion ($d < 1$) and in bold is the significant model chosen.

The initial set of results indicates that only the price of Dubai crude exhibits a “mean reversion” phenomenon among the ten variables studied. “Mean reversion” refers to a situation where a variable tends to return to its historical average over time, but in commodity markets, this is unexpected due to their short-term volatility. However, the case of Dubai is exceptional; perhaps due to the variety of factors that affect the supply and demand of crude oil in that market. For this reason, it is more likely that its price will approach its historical average. Notice that the d estimation of Natural gas in the US is 0.97, however, the upper limit of its confidence interval is greater than 1. This inhibits it from being a mean reverting process. For this reason, only the estimated d for the log prices in Dubai is mean reverting in Table 2. It must fulfill the condition that the estimated d is strictly below 1, including the upper value of its confidence interval.

Next, we look at the volatility by looking at the absolute and squared returns, obtained as the first differences in the log prices. With P_t being the price of the series of commodity markets, the first differences are obtained as $\log p_t - \log p_{t-1} = \Delta \log p_t \approx \frac{p_t - p_{t-1}}{p_{t-1}}$. The simple return is then $\frac{p_t - p_{t-1}}{p_{t-1}}$. Tables 3 and 4 display the same structure as Table 1 but for the absolute and squared returns respectively. Notice that the absolute return is $R_t = \left| \frac{p_t - p_{t-1}}{p_{t-1}} \right|$ and the squared return is $R_t^2 = \left[\frac{p_t - p_{t-1}}{p_{t-1}} \right]^2$. The absolute and squared returns are used as proxies of the volatility due to their capacity to capture changes in prices.

Starting with the absolute returns, in Table 3, we observe that all values exhibit mean reversion, as indicated by their d estimates and corresponding confidence intervals (CIs), which are all less than 1. Nevertheless, only the estimated values of d for the significant models, denoted in bold, are chosen econometrically. The significant model for most of the variables includes an intercept and a linear trend, except for Copper, Gold, and Silver, for which the model includes only an intercept. The economic interpretation is that since all values exhibit mean reversion, which is normal as volatility cannot exist infinitely, they are only periods of time caused by crises or global catastrophes. Therefore, if there is an economic shock capable of affecting commodity prices, volatility will tend to revert to the mean. There will not be many changes in volatility, and it will eventually disappear.

- When the value of d is close to 0, $d \approx 0$, meaning a small d , it will revert to the mean faster. In this case, among the significant models, those in which volatility will revert more quickly to zero are, Natural Gas EU, Natural Gas US, and Oil Dubai (0.15, 0.16 and 0.12, respectively).

- When the value of d is far from 0, meaning a slightly larger d , it will take a little longer to revert to the mean. In this case, it would be the case of all the other variables not previously mentioned (with the estimated d values are 0.27, 0.25, 0.23, 0.19, 0.20, 0.19 and 0.27, respectively).

But all of them do revert to the mean, whether fast or slow, they revert to the mean, to zero. Therefore, an economic shock will make volatility tend to disappear over time.

Continuing with the squared returns, in Table 4, similarly to the absolute returns case, there is evidence of mean reversion for all series in the squared returns as well. The significant models chosen for this analysis vary depending on the specific variable being considered. In the case of Oil Dubai and Silver, the chosen model is with no terms while for Oil Brent, Oil Average, Gold, Copper, and Aluminium, the chosen model is the one with only an intercept term. For Iron, Natural Gas EU, and Natural Gas US, the chosen model includes an intercept and a linear time trend.

It can be observed that Oil Dubai, Oil AV, Oil Brent, Natural Gas US, Iron, and Gold revert much faster to the mean than the other series. Their estimated memory parameters (d values) are 0.10, 0.05, 0.05, 0.09, 0.17, and 0.17, respectively. All squared return values revert to the mean, which means that if there is an economic shock, volatility will revert to the mean, and thus it would not affect the series, as it tends to disappear over time.

Table 3

Estimates of d for the absolute return series

Series	No terms	An intercept	An intercept and a linear time trend
Aluminium	0.26 (0.22, 0.31)	0.28 (0.24, 0.32)	0.27 (0.22, 0.31)
Copper	0.26 (0.21, 0.31)	0.25 (0.20, 0.30)	0.25 (0.20, 0.30)
Gold	0.23 (0.19, 0.27)	0.23 (0.20, 0.27)	0.24 (0.20, 0.28)
Iron	0.22 (0.18, 0.27)	0.23 (0.19, 0.28)	0.19 (0.14, 0.25)
Natural Gas EU	0.18 (0.15, 0.22)	0.19 (0.15, 0.23)	0.15 (0.11, 0.19)
Natural Gas US	0.21 (0.17, 0.25)	0.23 (0.19, 0.27)	0.16 (0.11, 0.21)
Oil AV	0.21 (0.17, 0.27)	0.23 (0.19, 0.28)	0.20 (0.15, 0.26)
Oil Brent	0.21 (0.16, 0.26)	0.22 (0.18, 0.27)	0.19 (0.14, 0.25)
OilDubai	0.12 (0.08, 0.17)	0.13 (0.08, 0.18)	0.12 (0.07, 0.17)
Silver	0.26 (0.11, 0.31)	0.27 (0.23, 0.31)	0.26 (0.23, 0.31)

There is evidence of mean reversion ($d < 1$) in all the results, but the significant model chosen is highlighted in bold.

Table 4

Estimates of d for the squared return series

Series	No terms	An intercept	An intercept and a linear time trend
Aluminium	0.24 (0.19, 0.29)	0.24 (0.20, 0.30)	0.24 (0.20, 0.30)
Copper	0.24 (0.19, 0.30)	0.24 (0.18, 0.30)	0.24 (0.18, 0.30)
Gold	0.17 (0.13, 0.21)	0.17 (0.13, 0.21)	0.17 (0.12, 0.21)
Iron	0.18 (0.12, 0.24)	0.18 (0.13, 0.25)	0.17 (0.11, 0.24)
Natural Gas EU	0.10 (0.06, 0.15)	0.10 (0.06, 0.15)	0.09 (0.05, 0.14)
Natural Gas US	0.21 (0.16, 0.28)	0.22 (0.16, 0.29)	0.19 (0.13, 0.27)
Oil AV	0.05 (0.00, 0.11)	0.05 (0.00, 0.11)	0.05 (0.00, 0.11)
Oil Brent	0.05 (0.00, 0.10)	0.05 (0.00, 0.10)	0.04 (0.00, 0.10)
OilDubai	0.10 (0.06, 0.15)	0.10 (0.06, 0.15)	0.10 (0.05, 0.15)
Silver	0.25 (0.21, 0.30)	0.25 (0.21, 0.30)	0.25 (0.21, 0.30)

There is evidence of mean reversion ($d < 1$) in all the results, but the significant model chosen is highlighted in bold.

Before conducting the empirical test on fractional cointegration, it is important to analyze the memory parameter as we did previously. If all the variables are non-stationary, as is the case with the commodity prices (Table 2), it will be possible to use the FCVAR approach. By using the AIC information criterion, the lag, $k=2$, seems to be suitable for the estimation of a FCVAR with two lags in the model and to test for cointegration relationships among the variables. As we can see in the results provided in Table 5, there are four fractional cointegration relationships among the 10 commodity prices.

Table 5

Likelihood Ratio Tests for Fractional Cointegration

Rank	d	b	Log-likelihood	LR statistic	P-value
0	0.605	0.605	-21046.551	399.099	0.000
1	0.529	0.529	-20950.114	206.224	0.000
2	0.560	0.560	-20921.715	149.425	0.000
3	0.604	0.604	-20898.455	102.906	0.002
4	0.639	0.639	-20880.500	66.997	0.068
5	0.629	0.629	-20867.156	40.309	0.275
6	0.513	0.513	-20857.006	20.009	0.492
7	0.499	0.499	-20851.461	8.918	0.445
8	0.518	0.518	-20848.184	2.364	0.893
9	0.504	0.504	-20847.084	0.164	0.685
10	0.519	0.519	-20847.002	----	----

There is evidence that we cannot reject the null hypothesis of the existence of four cointegrating relationships at a significance level of 5%.

6. CONCLUSIONS

This paper was motivated by the study of volatility and long-run relationships among the worldwide commodity markets. We explored monthly data from January 1960 to September 2022 on ten important commodity prices. First, we conducted an analysis of the raw log prices of these commodities to check their level of integration, finding only one variable that reverts to the mean (crude Dubai oil). Indeed, we find that only the Dubai oil price reverts to the mean. This suggests that Dubai is a wealthy and stable economy with a long reserve of oil, making its production less vulnerable to fluctuations in the global market and less sensitive to economic crises and price volatility, which accurately reflects the reality of the situation. Second, in response to the objective of this paper, which investigates the progression of volatility over time in commodity markets, we can conclude that both proxies of volatility used for our sample are reverting to the mean significantly. Some variables revert to the mean quickly (when d is near to zero), while others do not (where d is far from zero). However, in both proxies, the reversion to the mean signifies that volatility cannot persist forever and is only present during periods of crises or worldwide disasters. Over time, the volatility will disappear and revert to zero. This indicates that a crisis is not permanent, as they come and go depending on the circumstances. Therefore, the volatility results accurately reflect reality.

Regarding long-term relationships, our findings reveal the presence of four cointegrating relationships among the ten commodity prices. This suggests that these commodities are not entirely independent entities; rather, they appear to be interconnected, sharing underlying relationships that contribute to their price fluctuations over time. This significant insight enhances our understanding of the intricate interplays within the commodity market.

Finally, our findings suggest that volatile series tend to dissipate and revert to their mean levels, as discussed previously, even in the absence of effective policy measures. This finding is consistent with the principle of mean reversion, which holds that price levels tend to return to their long-run average values over time. Hence, while policy interventions may help to mitigate volatility in the short term, over the long term, natural market forces tend to bring prices back to their equilibrium levels.

This prompts us to reflect on what effective policies could be implemented to mitigate volatility. What measures should certain countries implement to prevent volatility from increasing gradually? It is crucial to have strong and sustainable macroeconomic policies to mitigate volatility in commodity prices. The following key ingredients could be needed:

(i) Strong fiscal frameworks that encourage counter-cyclical fiscal policy, meaning saving during good times to spend during bad times.

(ii) Exchange rate flexibility linked to a monetary policy with low inflation objectives.

(iii) A regulatory system for the financial sector that prevents the accumulation of excessive risks, particularly related to capital inflows and foreign currency debt.

In other words, having a plan for managing money during both good and bad times, controlling inflation, regulating financial systems, and using tools to manage risk can help to mitigate volatility in the short term against unpredictable changes in commodity prices. It is worth considering that crises such as the Russian-Ukrainian war and the COVID-19 pandemic could have been better mitigated with effective policies from the beginning, thus avoiding the high inflation rates currently being experienced worldwide. Although volatility tends to revert to the mean, it does not happen in the short term, leading to the first-hand experience of high inflation caused by these global catastrophes. These facts reflect the observed patterns in the 1960s and 1970s, thereby highlighting the crucial significance of strategic planning and implementing policies to effectively mitigate the adverse effects of such crises.

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