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Autoregression of Bilateral Trade Flow Matrices

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Abstract

Amongst a considered set of trading partners, does proportional bilateral trade flow in matrix form have autoregressive predictability? In this paper, the standard method of ordinary least squares (OLS) regression is modified for the input and prediction of matrices representing proportional exports of trading partners. Considering a small sample of trading partners and using the matrix regression model proposed, we find some evidence of autoregressive predictability. This predictability can be employed to predict future trade flows among different countries. Our findings provide some potential policy implications on international trade.

Keywords: international trade; trade flow; autoregressive predictability, etc.

Introduction

Prior studies on the determinants of trade focus primarily on the relationship between distance and trade (Srivastava and Green, 1986). One of the most well-known models to consider is the linear regression-based gravity model (Anderson, 1979). In reality, the relationship may not be linear.

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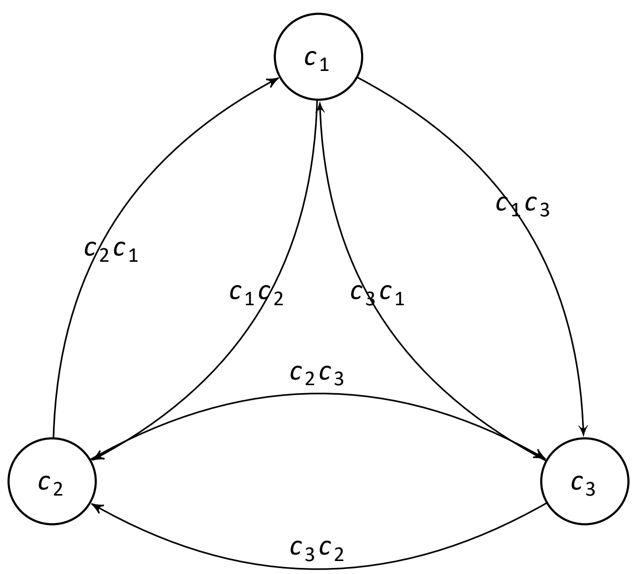
Exploring international crude oil data and the link prediction approach in Lu and Zhou (2011), Guan et al. (2006) estimate the potential trade links using a link prediction approach. They identify the number of common trade partners as one of the structural linking motivations. It also reflects the competition between nations. They estimate this potential trade partner combined with nations' crude oil trade roles. Moreover, to achieve more accurate predictions, research has been conducted into alternative machine learning methods for trade flow prediction (Gupta and Kumar, 2021).

Amongst the methods studied, include the ARIMA model (Batarseh et al., 2019) and neural network analysis (Wohl and Kennedy, 2018, Circlaeys et al., 2017). This paper aims to analyze the autoregressive predictability of global trade between countries using trade flow data. The trade flow of several partners over two units of time is represented as a Markov chain, and we will introduce a model to forecast future trade flows denoted as a Markov chain. In this paper, some set of trading partners is considered and denoted as C. With this, the export activity from some partner c_a to some partner c_b over time range $[t - 1, t]$, denoted as $c_a c_b$, is measured as the proportion of the export value of c_a going to c_b out of the total export value going from c_a to trading partners in C. This allows for trade activity over a certain time range to be presented as a Markov chain



and thus a transition matrix

$$y_t = \begin{bmatrix} 0 & c1c2 & c1c3 \\ c2c1 & 0 & c2c3 \\ c3c1 & c3c2 & 0 \end{bmatrix}_t$$

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The transition matrix y_t is displayed where transitions occur going from row to column as it represents proportional export activity amongst trading partners in C over a time range $[t-1, t]$. With this, the task is to design a model that can approximate y_t using proportional export flow matrices corresponding to previous years. This setup and visualization resemble the task of predicting trade flows using link prediction algorithms Vidmer et al., 2015. However, in this case, industry corresponding to the trade flow is irrelevant, and the existence of a link between any two partners in C *is* assumed to always exist. Still, it can have a value of zero to represent no trade flow.

Methodology

Prediction Model

All samples of y_t will be $n \times n$ matrices as C contains n elements, and we will use l autoregressive independent variables. The structure of the model for an ordinary least squares (OLS) regression will be used, but with the input variables as well as the output being matrices

$$f(x) = \hat{y}_t = \underbrace{\beta_0}_{\text{intercept}} + \underbrace{\sum_{i=1}^{l} \beta_i y_{t-i}}.$$

weighted sum

The loss function used to train the model is

$$\mathcal{L}(\vec{\beta}) = \frac{1}{m} \sum_{k=1}^{m} \frac{1}{n^2} \sum_{j=1}^{n} \sum_{i=1}^{n} (\hat{y}_{ij} - y_{ij})^2$$

where it computes the average of all mean sum of squared differences of matrix entries for each true and predicted matrix over all samples.

Optimization

With the loss function defined, the update method for the optimizable parameters is

$$\vec{\beta} \leftarrow \vec{\beta} - \gamma \nabla \mathcal{L} = [\beta_0, \ldots, \beta_n] - \gamma [\frac{\partial \mathcal{L}}{\partial \beta_0}, \ldots, \frac{\partial \mathcal{L}}{\partial \beta_n}]$$

for some learning rate γ. Considering some integer g where $0 < g \leq l$ and letting $y_{t-g} = x$, taking the partial derivative of the loss function with respect to some arbitrary parameter β_g gives

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$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \beta_g} &= \frac{\partial}{\partial \beta_g}\left(\frac{1}{m}\sum_{k=1}^{m}\frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}(\hat{y}_{ijk}-y_{ijk})^2\right)\\ &= \sum_{k=1}^{m}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{2x_{ijk}}{mn^2}(\hat{y}_{ijk}-y_{ijk})^2.\end{aligned}$$

To summarize, the algorithm used the train the optimizable parameters is as follows.

Since the linear least squares regression model is being used for matrix variables, the R^2 coefficient is now computed as

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum_{k=1}^{m}\sum_{j=1}^{n}\sum_{i=1}^{n}\left(y_{ijk}-\hat{y}_{ijk}\right)^2}{\sum_{k=1}^{m}\sum_{j=1}^{n}\sum_{i=1}^{n}\left(y_{ijk}-\bar{y}_{ij}\right)^2}$$

with

$$\bar{y} = \frac{1}{m}\sum_{i=1}^{m} y_i$$

Algorithm 1 Parameter optimization

$\vec{\beta} \leftarrow$ random values

$A \leftarrow 0$
while $A \neq$ epochs **do**

$\beta = \beta - \gamma \nabla L$

$A = A + 1$
end end while

Data

We collect the data on international trade from the Atlas of Economic Complexity by the Growth Lab at Harvard University[1]. The data contains record of bilateral trade flow across nations over time and also specifies the corresponding industry of trade flows.

[1] https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/H8SFD2

Empirical Results

To perform the analysis, a sample of four countries was considered. This set of countries consisted of the USA, China, Japan, and Germany. For this experiment, the intercept parameter of the model was not included so as to make the diagonal values of the predicted transition matrices nonzero. With the methodology carried out using six autoregressive variables and sixty epochs, the optimizable parameters of the model converged to the following values.

Table 1: Resulting parameter values

Variable	β
$yt-1$	0.19703017
$yt-2$	-0.3062691
$yt-3$	0.49183621
$yt-4$	0.29299555
$yt-5$	0.47389675
$yt-6$	0.00679866

The ending R^2 and loss values were 0.5702 and 0.0063, respectively. To visually compare the predicted and true values, plots were made where the true values are shown in solid lines, and the predicted values are shown in dashed lines.

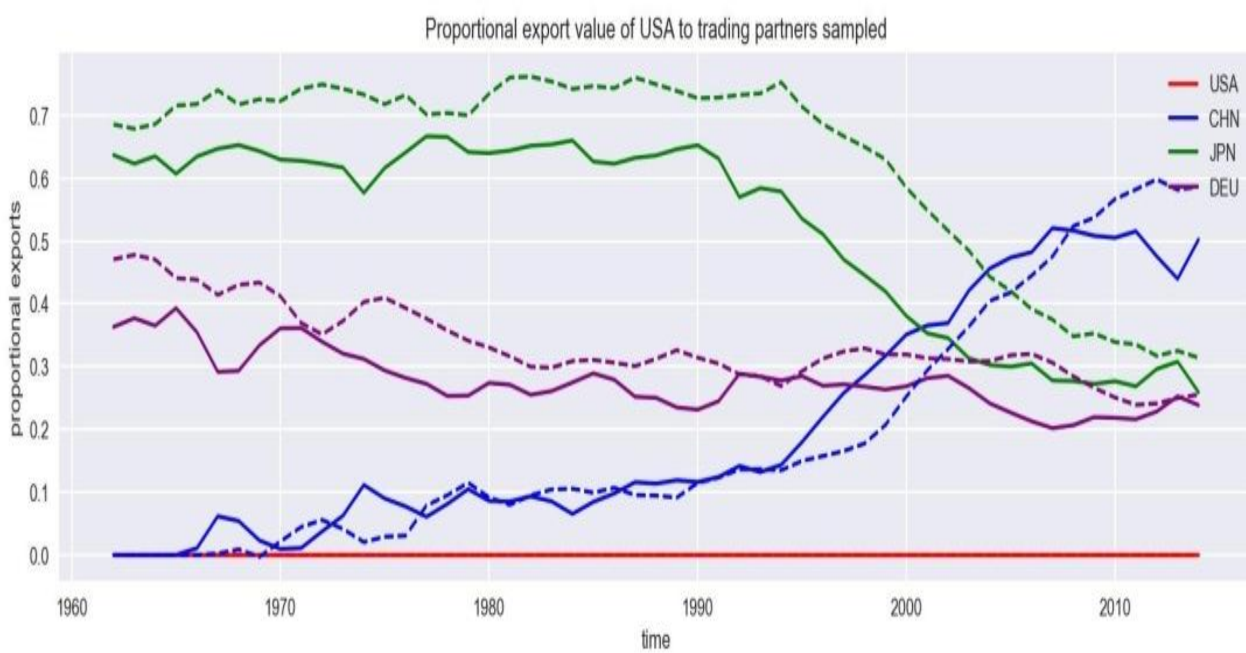

Figure 1: True and predicted proportional exports of the USA

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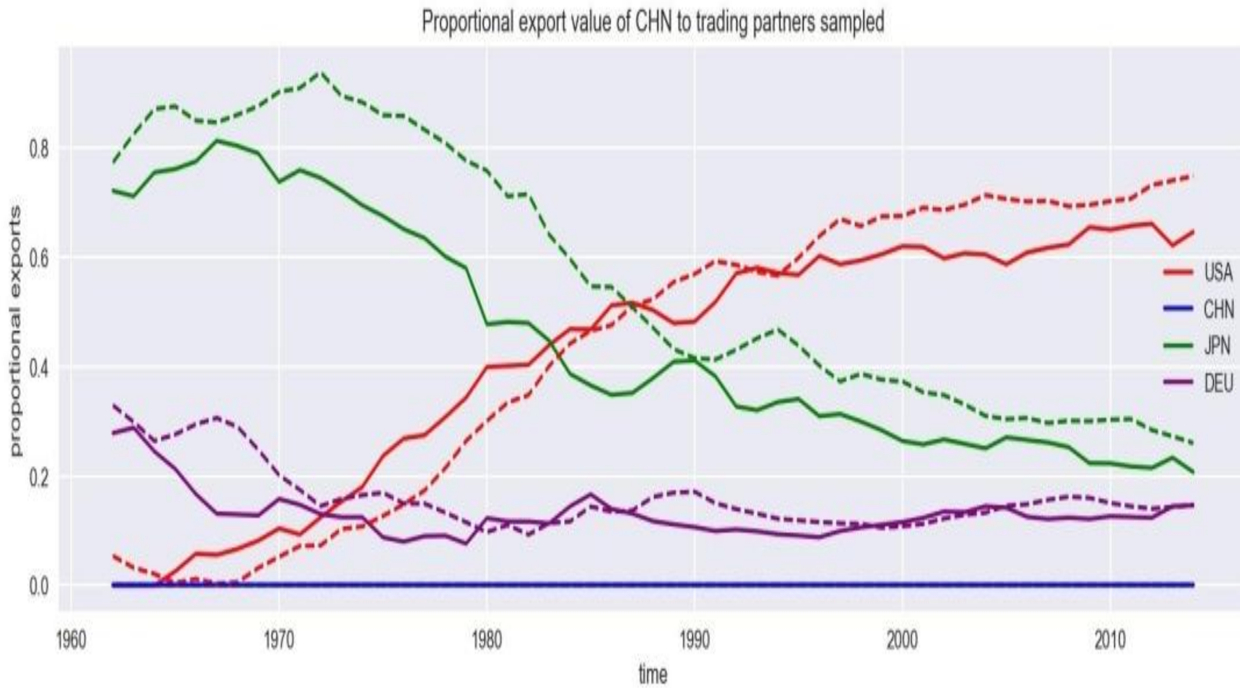

Figure 2: True and predicted proportional exports of CHN

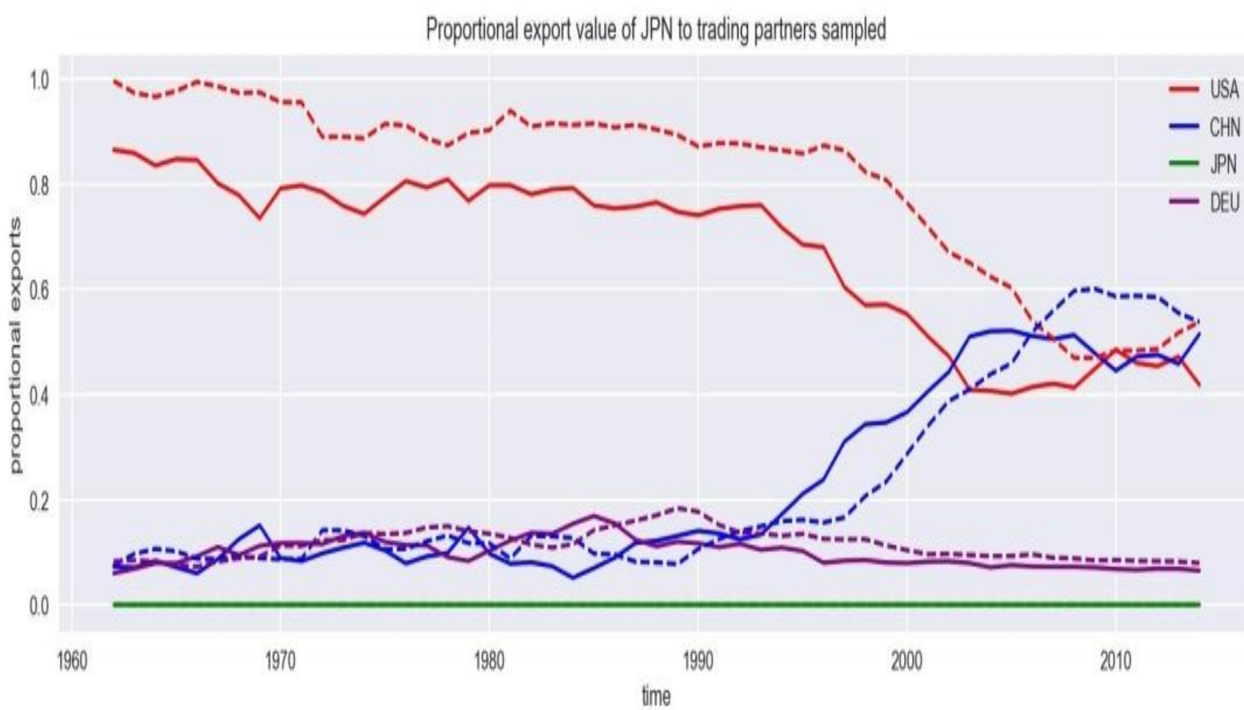

Figure 3: True and predicted proportional exports of JPN

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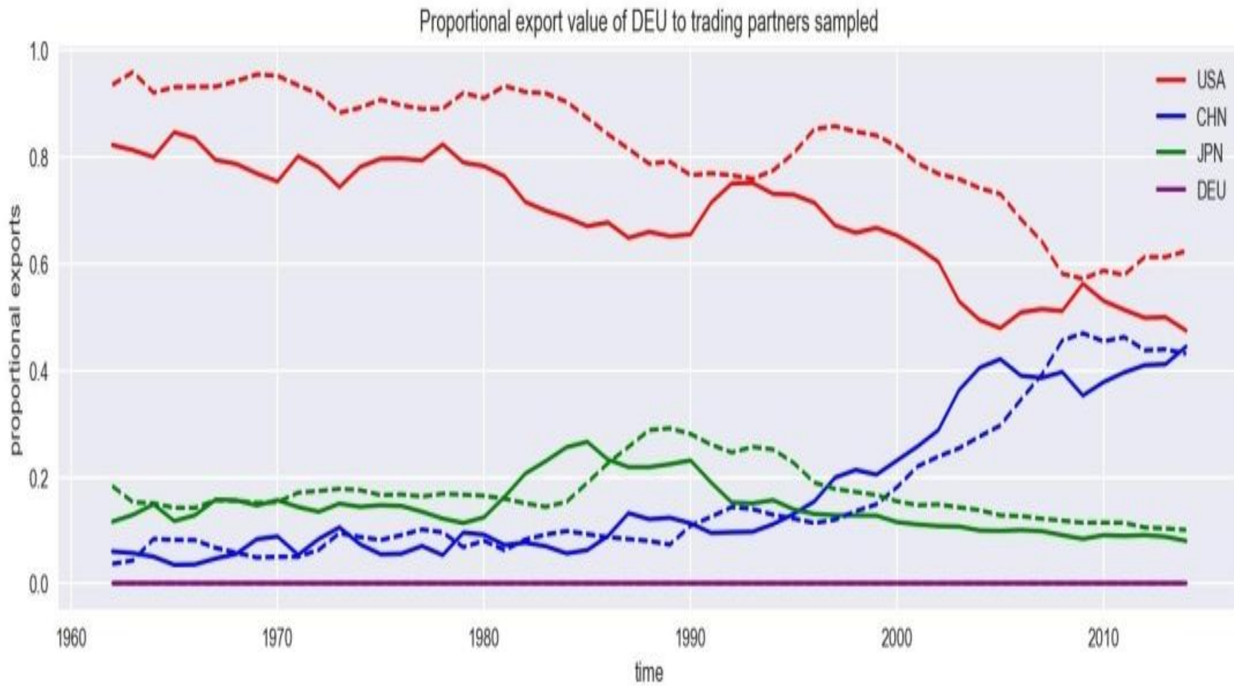

Figure 4: True and predicted proportional exports of DEU

Conclusion

Given the empirical results, yearly proportional exports denoted in matrix form have some linear autoregressive predictability. Though analysis with this model has shown to provide a decent amount of accuracy, its significance regarding any policy implications does not seem to present itself clearly as has been the case for other work (Yu, 2010). Considering this model as a starting point to be expanded upon to include the related industry of trade flow and other factors pertaining to trade partners may prove to have meaningful policy implications.

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